The 33rd Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 22nd April 2026
Category I

Problem 1 *Let n be an odd integer and let A and B be complex $n \times n$ matrices such that $A, B, A - B$ are invertible and*

$$(A - B)^{-1} = A^{-1} - B^{-1}.$$

Prove that

$$\det A \det B = -(\det(A - B))^2.$$

[10 points]

Problem 2 *Géza has invited his $n \geq 2$ friends to dinner. At his round table there are exactly n seats placed equidistantly, and Géza placed name cards on the seats that completely determine the seating arrangement. Unfortunately, the friends ignored the name cards and just sat down arbitrarily.*

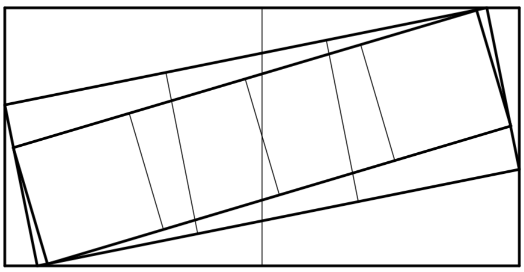
Depending on n, find the largest possible positive integer $k(n)$ with the following property: For any seating arrangement, Géza can rotate the table so that after the rotation at least $k(n)$ friends are seated correctly.

(A rotation by 0 positions is also considered a rotation.) [10 points]

Problem 3 *Let us start with a rectangle R_2 consisting of two unit squares. Let us consider*

- *a rectangle R_3 – consisting of three identical squares in a row – inscribed in R_2 (this means that vertices of R_3 lie on distinct sides of R_2),*
- *a rectangle R_4 – consisting of four identical squares in a row – inscribed in R_3,*
- *...*
- *a rectangle R_{n+1} – consisting of $n + 1$ identical squares in a row – inscribed in R_n, and so on.*

Is it possible to put all rectangles next to each other in a row on a finite desk?
In other words: Denoting by a_n the length of the shorter side of R_n, decide whether $\sum_{n=2}^{\infty} a_n$ converges.



[10 points]

Problem 4 *Let $f : \mathbb{R} \to \mathbb{R}$ be a bounded continuous function. Suppose that there exists a positive number a such that*

$$g(x) = f(x) + a \int_{x-1}^{x} f(t)\, \mathrm{d}t, \quad x \in \mathbb{R},$$

is a constant function. Show that f is a constant function. [10 points]