The 23rd Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 12th April 2013
Category II

Problem 1 *Let S_n denote the sum of the first n prime numbers. Prove that for any n there exists the square of an integer between S_n and S_{n+1}.*

Problem 2 *An n-dimensional cube is given. Consider all the segments connecting any two different vertices of the cube. How many distinct intersection points do these segments have (excluding the vertices)?*

Problem 3 *Prove that there is no polynomial P with integer coefficients such that $P(\sqrt[3]{5} + \sqrt[3]{25}) = 5 + \sqrt[3]{5}$.*

Problem 4 *Let $\mathcal{F}$ be the set of all continuous functions $f\colon [0,1] \to \mathbb{R}$ with the property*

$$\left| \int_0^x \frac{f(t)}{\sqrt{x-t}} \, \mathrm{d}t \right| \le 1 \quad \text{for all } x \in (0,1]\,.$$

Compute $\sup_{f \in \mathcal{F}} \left| \int_0^1 f(x) \, \mathrm{d}x \right|$.