The 6th Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 3rd April 1996
Category I

Problem 1 *On the ellipse* $\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$ *find the point* $T = [x_0, z_0]$ *such that the triangle bounded by the axes of the ellipse and the tangent at that point has the least area.*

Problem 2 *Let* $\{a_n\}_{n=0}^{\infty}$ *be the sequence of integers such that* $a_0 = 1, a_1 = 1, a_{n+2} = 2a_{n+1} - 2a_n$. *Decide weather*

$$a_n = \sum_{k=0}^{[n/2]} \binom{n}{2k}.$$

Problem 3 *Prove that the equation*

$$\frac{z}{1+z^2} + \frac{y}{1+y^2} + \frac{x}{1+x^2} = \frac{1}{1996}$$

has finitely many solutions in positive integers.

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Category II

Problem 1 *Is it possible to cover the plane with the interiors of a finite number of parabolas?*

Problem 2 *Let* $\{x_n\}_{n=0}^{\infty}$ *be the sequence such that* $x_0 = 2, x_1 = 1$ *and* x_{n+2} *is the remainder of the number* $x_{n+1} + x_n$ *divided by* 7*. Prove that* x_n *is the remainder of the number*

$$4^n \sum_{k=0}^{[n/2]} 2\binom{n}{2k} 5^k$$

divided by 7*.*

Problem 3 *Let* $\operatorname{cif}(x)$ *denote the sum of the digits of the number* x *in the decimal system. Put* $a_1 = 1997^{1996^{1997}}$, $a_{n+1} = \operatorname{cif}(a_n)$ *for every* $n > 0$*. Find* $\lim_{n\to\infty} a_n$*.*