The 5^{th} Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 25^{th} – 26^{th} April 1995
Category I

Problem 1 *Discuss the solvability of the equations*

$$\begin{aligned}\lambda x + y + z &= a\\ x + \lambda y + z &= b\\ x + y + \lambda z &= c\end{aligned}$$

for all numbers $\lambda, a, b, c \in \mathbb{R}$.

Problem 2 *Let* $f(x)$ *be an even twice differentiable function such that* $f''(0) \neq 0$. *Prove that* $f(x)$ *has a local extremum at* $x = 0$.

Problem 3 *Let* $f(x)$ *and* $g(x)$ *be mutually inverse decreasing functions on the interval* $(0, \infty)$. *Can it hold that* $f(x) > g(x)$ *for all* $x \in (0, \infty)$?

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Category II

Problem 1 *Prove that the systems of hyperbolas*

$$x^2 - y^2 = a \qquad (1)$$
$$xy = b \qquad (2)$$

are orthogonal.

Problem 2 *Let* $f = f_0 + f_1 z + f_2 z^2 + \ldots + f_{2n} z^{2n}$ *and* $f_k = f_{2n-k}$ *for each* k. *Prove that* $f(z) = z^n g(z + z^{-1})$, *where* g *is a polynomial of degree* n.

Problem 3 *Let* $f \colon \mathbb{R} \to \mathbb{R}$ *be a continuous function. Do there exist continuous functions* $g \colon \mathbb{R} \to \mathbb{R}$ *and* $h \colon \mathbb{R} \to \mathbb{R}$ *such that* $f(x) = g(x) \sin x + h(x) \cos x$ *holds for every* $x \in \mathbb{R}$?

Problem 4 *Let* $\{x_n\}_{n=1}^{\infty}$ *be a sequence such that* $x_1 = 25$, $x_n = \arctan x_{n-1}$. *Prove that this sequence has a limit and find it.*