The 2$^{\text{nd}}$ Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 28$^{\text{th}}$ April 1992
Category I

Problem 1 *Find the* n^{th} *derivation of the function*

$$f(x) = \frac{x}{x^2 - 1}\,.$$

Problem 2 *Prove that there exist two real convex functions* f, g *such that*

$$f(x) - g(x) = \sin x$$

for all $x \in \mathbb{R}$.

Problem 3 *Prove that for all integers* $n > 1$,

$$(n-1)|(n^n - n^2 + n - 1)\,.$$

Problem 4 *Let* X *be a finite set and* $f\colon X \to X$ *be map. Prove that* f *is an injective map if and only if* f *is a surjective map.*

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Category II

Problem 1 *Prove that for a continuously differentiable function $f(x)$, where $f(a) = f(b) = 0$,*

$$\max_{x\in[a,b]} |f'(x)| \geq \frac{1}{(b-a)^2} \int_a^b |f(x)| \, \mathrm{d}x \, .$$

Problem 2 *Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ which satisfy the equality*

$$xf(y) + yf(x) = (x+y)f(x)f(y) \, .$$

Problem 3 *Let Z_k be the additive group of residual classes modulo k. Decide if Z_6 is isomorphic to $Z_2 \times Z_3$.*

Problem 4 *Prove that each rational number $\frac{p}{q} \neq 0$ can be written in the form*

$$\frac{p}{q} = b_1 + \frac{b_2}{2!} + \cdots + \frac{b_n}{n!} \, ,$$

where n is a sufficiently large positive integer and $b_k \in \mathbb{Z}$ $(k > 1)$ such that $0 \leq b_k < k,\ b_n \neq 0$.